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Use of Artificial Satellites for Navigation and Oceanographic Surveys

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Technical Bulletin Series

This series of Technical Bulletins was inaugurated to present primarily to the personnel of the Coast and Geodetic Survey and incidentally to others technical information related to the Bureau's scientific and technical activities. Since many of the bulletins deal with new practices and new techniques, the views expressed are those of the authors and do not necessarily represent final Bureau policy.

Technical Bulletin No. 12 describes the use of near-earth satellites in the field of navigation and oceanographic surveys. It is a companion to Technical Bulletin No. 11. The subject has been developed against a background of terrestrial navigational systems both celestial and electronic. The mathematics used is found in three appendixes. The first, on the relationship between satellite velocities, periods, and heights, was written by Mr. Lansing G. Simmons.

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Besides his method of determining the longitude from Jupiter's satellites, Galileo's studies of the pendulum and its behavior led to the attachment of the pendulum to a clock mechanism as a driving force by Christian Huygens (son of Constantine Huygens who had known Galileo) in 1656. The stimulus of the longitude problem and the growth of interest in science in general throughout Europe provoked simultaneously the transition of cartography from an art to a science. Maps should be based on positions of known latitude and longitude, and in 1666, the Academie Royale des Sciences (now the Institut of France) was established by King Louis XIV ostensibly for the purpose of research in producing authentic maps and sailing charts. Since astronomy was the prime science involved, it received the first impetus from the Academie. Improvements were made in the telescope, the effects of gravity on the pendulum were studied, and the results helped Huygens perfect his pendulum timekeeper. The shape and size of the earth were studied (particularly the problem of establishing the linear value of a degree of longitude which would be accepted universally). Investigations were made of the motions of the moon, earth, and other celestial bodies. Methods of determining the longitude were exhaustively studied and, in 1669, the scientists of the Academie turned again to the four largest moons of Jupiter (a half century after Galileo's proposal!). Of course additional information had been compiled in the long interval. Cassini (an Italian astronomer, a member of the Academie, who became a naturalized French subject in 1673), had studied Jupiter's moon system, using fairly reliable pendulum clocks and an improved telescope. He published extensive tables (ephemerides) of the eclipses of Jupiter's satellites for the year 1668. But no method of determining longitude would be useful for navigation if the size of the circumference of the earth were not known accurately enough. After an exhaustive study by members of the Academie of the measurements of Hipparchus, Ptolemy, Snell, and others, it was decided to make a new determination and Jean Picard (a French mathematician) was assigned the task in 1669. An arc of the meridian was to be measured between two assigned points (on the same meridian) by means of a network of triangles running north and south between the terminals. By astronomical observations at the terminals the latitude difference would be obtained. Picard used telescopes with oculars fitted with crosshairs in his angle measuring instruments, so the small net of 13 triangles connecting the Pavillon at Molvoisine near Paris with the clock tower in Sourdon near Amiens was probably the first modern geodetic triangulation (from the standpoint of using reticles in telescopes). From this determination the diameter of the earth was found to be an equivalent of 7,801 statute miles, a remarkably close result.

With a new value of the earth's circumference and a method of determining the longitude by observations of Jupiter's satellites, the Academie lost no time in sending expeditions to various parts of the earth to determine their longitudes, which were referred to the meridian of Paris. The method is simple enough—to find the difference in local or mean time between the prime meridian of Paris and a second place whose longitude is required, the difference in time being equivalent to the difference in longitude. Two pendulum clocks were used, one adjusted to keep mean time--24 hours a day. The second clock was set to keep sidereal or star time ($23^h56^m4^s$). With a meridian line established and latitude determined from solar and Pole-Star observations, the time of the eclipses of the four largest satellites of Jupiter were observed, at least two of which are eclipsed every 2 days. By use of the prepared ephemerides of the times of eclipse at Paris the difference in local or mean time could be determined and hence the longitude.

Some of these expeditions, those near the equator, reported strange behavior in their pendulum clocks. In order to get them to keep mean time they had to shorten the pendulums (raise the bobs). Cassini supposed that it was error of observation, but Sir Isaac Newton, who had been making a theoretical investigation of the shape of the earth based on Huygen's publication in 1673 on the oscillation of the pendulum (where is also found the first sound theory of centrifugal force), was elated. In the third book of his ***Principia*** he concluded that this variation of the pendulum in the vicinity of the equator must be caused either by a diminution of gravity resulting from a bulging of the earth at the equator or from the strong, counteracting effect of centrifugal force in that region. By 1673, the evidence had been presented that the earth was slightly flattened at the poles, but not to be accepted universally even by all the scientists of the day—not until the results were reported of expeditions which had been sent to Peru and Lapland, in 1735 and 1736, to measure arcs of the meridian.

It was early realized that the method of using the satellites of Jupiter to determine longitude was not practical for use at sea. In 1675 England established the Royal Observatory for the advancement of navigation (search for the longitude) and nautical astronomy in Greenwich Park, overlooking the Thames River and the plain of Essex. While many crackpot schemes were proposed, there were some practical proposals and in 1714 Newton wrote "that for determining the longitude at sea, there have been several projects, true in theory, but difficult to execute . . . One

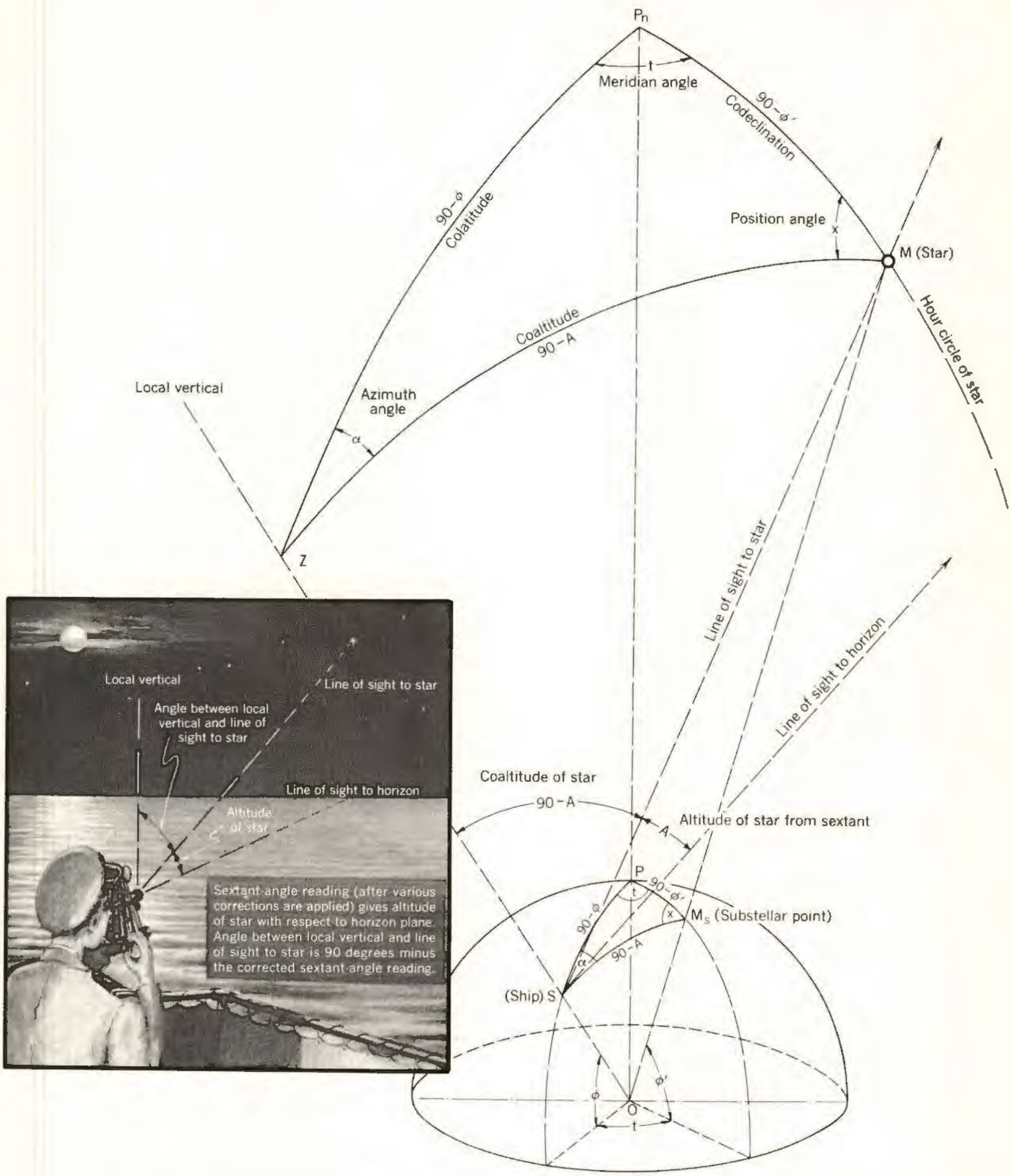

FIG. 1.--The Navigational Triangle

The stars are considered as lying on the surface of a sphere, the "celestial sphere," and the spherical triangle $Z\ P_n\ M$, where Z and P_n are the projections of the local vertical and polar axis of the earth on the celestial sphere, is called the Navigational Triangle. The terrestrial triangle $S\ P\ M_S$ may also be called the Navigational Triangle.

is by a watch to keep time exactly . . . such a watch hath not yet been made." In the same year the British Parliament passed a bill authorizing the payment of 20,000 English pounds for any device that would determine the longitude within 30 minutes (2 minutes of time or 34 miles). For 50 years this prize lay untouched—the problem of the longitude had stopped the best minds in Europe including Newton, Halley, Huygens, Leibnitz, and the rest. It was solved and the prize won by John Harrison, the son of a Yorkshire carpenter. His device was an accurate clock, the marine chronometer, finally perfected in 1772. Thus, longitudes were computed with reference to the meridian of the Royal Observatory at Greenwich, which was officially established as the prime meridian in 1884 by action of 25 nations at an international conference in Washington.

Figure 1 shows the basic navigational spherical triangle. One side of this spherical triangle is measured directly by use of the sextant as shown. (Since the stars are infinitely distant as compared to the radius of the earth, for practical purposes the point of observation is considered as coincident with the center of the earth.) If the particular star being observed is known, its codeclination or colatitude is known from a nautical almanac. Hence, two sides of the triangle are known. Also when the observations on the stars are made, the exact G.M.T. (Greenwich mean time) is known from the accurate shipborne chronometer. (The error of the chronometer is determined from daily radio time signals.) Since the almanac will give the G.M.T. when the particular star is in the meridian of Greenwich, the difference in G.M.T. times will give the hour angle or the longitude of the star. Hence, one has the geographical coordinates of M_S, the substellar point, and the zenith distance 90°-A, so the ship lies on a circle of radius 90°-A, about M_S as center. This is called a position circle. Note that ***a single stellar observation leads only to a small circle of position on which the ship is located.*** If a second observation is made simultaneously on a second celestial body, at the same G.M.T., then a second circle of position results and the ship is at one of the two points of intersection of these two circles (see fig. 2). Since the approximate position of the ship is usually known by dead reckoning (a continuous graphical accounting of the ship's progress, knowing the ship's course and speed), the proper point of intersection (the ship's true position) is usually easily ascertained. Basically this is the essence of the problem and many short methods using tabular data and charts have been devised. Since the radius of a circle of position may be too large to use on a chart, a small portion of the circle (or its tangent) in the vicinity of the observer may be used, the intersection of two such tangents to two circles of position, or Sumner lines giving a fix (see fig. 2). If the same or another celestial object is used to obtain two separate lines of position after a lapse of time, the first line being advanced to the line of the second (by dead reckoning), the fix obtained is called a running fix. The sun is often used for this purpose. The actual practice of celestial navigation has become a refined art.[1] Indeed a proposed form of automatic celestial navigation uses the principle of the planetarium in reverse—two bodies serving to position a horizontal-stabilized sphere (in principle) for latitude and local sidereal time. If the device is accurately set to Greenwich sidereal time, longitude is indicated.

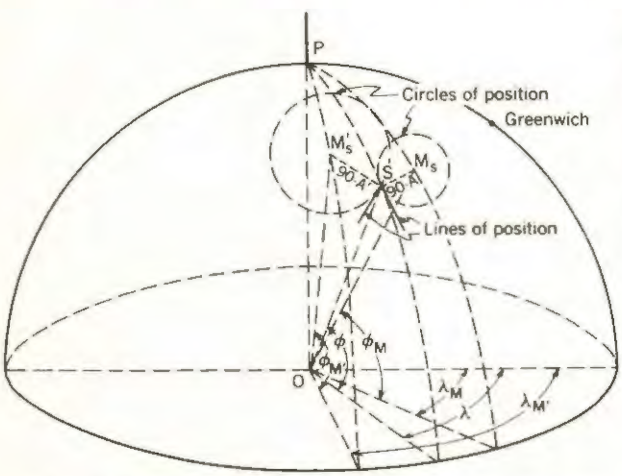

FIG. 2.—Position as sea from the intersection of two position circles.

If the stars M and M' are known their latitudes are known from the almanac. Their longitudes may be determined by the difference in G.M.T. of the observation from the almanac listed G.M.T. for each star. Hence the geographic positions of the substellar points $M_S(\phi_M, \lambda_M)$ and $M_S'(\phi_{M'}, \lambda_{M'})$ are known and the distances $SM_S = 90° - A$, $SM_S' = 90° - A'$ are measured with the sextant. Hence the ship S lies at one of the intersections of the small circles about M_S and M_S' of radii $90° - A$ and $90° - A'$, respectively.

RADIO, RADAR, ELECTRONIC POSITIONING SYSTEMS

Radio and its higher frequency counterpart, radar, revolutionized the old art of contact navigation or position fixing by landmarks. Radio waves provided the means of contact, with land-based or ship-based transmitters making possible not only contacts day or night over long distances beyond human visual capability, but also under weather conditions which would preclude visual contact.

[1]See American Practical Navigator, Bowditch, *U. S. Navy Hydrographic Office*, 1958 ed.

The Scottish mathematical physicist, James Clerk Maxwell, predicted electromagnetic waves in space in his electromagnetic theory of light which he developed between 1867 and 1873. But it was not until 1887 that the German physicist, Heinrich Hertz, first produced "Hertzian" or radio waves in the laboratory. In 1894, the Italian physicist, Guglielmo Marconi, began experimenting with transmission and receipt of radio signals. He was familiar with the work of Maxwell, Hertz, and others. With little encouragement in Italy he went to England in 1896. There he received every assistance, filing his first patent in June 1896. He steadily improved the range of his equipment, achieving his greatest triumph in 1901. Besides general skepticism concerning the usefulness of this "wireless" type of communication, some distinguished mathematicians of the day had stated that the curvature of the earth would limit practical communications by means of radio waves to a distance of 100 to 200 miles. But Marconi succeeded in December 1901 in receiving at St. Johns, Newfoundland, signals transmitted across the Atlantic Ocean from Poldhu in Cornwall. This may be considered the starting point of the vast development of radio communications, broadcasting and navigation services that took place in the next 50 years and in which Marconi continued to play an important part until his death in 1937.

Radio waves are physically similar to light waves. Both are electromagnetic radiations having the same speed of propagation in free space of 299,792.5 km./sec. There are two differences—the wave lengths of radio waves are much longer than those of light, and, unlike light waves, man-made radio waves are coherent in phase, which permits resonance—that is accumulation of energy over several cycles in a synchronized receiver. Radio waves range in length from 6 mm. to 6 miles, and the shortest is 10,000 times the wave length of light!

Hertz found in 1887 that radio waves were reflected from solid objects and investigation of the phenomenon was continued through the years. In fact a German engineer, Christian Hulsmeyer, was granted patents in several countries in 1904 on a radio-echo collision-preventive device. In June 1922, Marconi in a speech before the Institute of Radio Engineers in New York said, "It seems to me that it should be possible to design apparatus by means of which a ship could radiate or project a divergent beam of these rays in any desired direction, which rays, if coming across a metallic object, such as another steamer or ship, would be reflected back to a receiver screened from the local transmitter on the sending ship, and thereby immediately reveal the presence and bearing of the other ship in fog or thick weather."

In 1925, Gregory Breit and Merle A. Tuve of the Carnegie Institution of Washington, D. C., in a series of experiments designed to measure the height of the ionosphere, used the pulse principle almost exactly in the form in which it came to be used in radar (RAdio Detection And Ranging). Successful pulse radar systems were developed independently and nearly simultaneously in the United States, France, England, and Germany during the years 1935-1940. By 1940, the multi-cavity magnetron (a type of transmitting tube developed specifically for radar use) had been developed to the point where it gave about 10 kw. of pulse power. This made microwave radar practical for the first time, and modern radar can be said to date from this device. The impetus of World War II resulted in the development of radar systems for many purposes (by and for the military establishments). In general these systems may be classed as primary and secondary. In primary radar, a reflected signal or echo is returned and displayed on the face of a cathode ray tube or oscilloscope. In secondary radar, the transmitted signal serves as an interrogator to trigger a transponder, which immediately (or after a known delay) transmits a return signal. The latter is the basic principle of equipment such as shoran, hiran, and the Coast Survey Electronic Position Indicator. The primary radar enables a ship literally to see all objects radially about it, in its vicinity, at night, in fog or adverse weather. Since it gives automatically the bearing and distance to the object (the distance is obtained by multiplying half the time required for the radio waves to reach the object and return to the ship by the velocity of radio waves—the speed of light) radar is of invaluable aid in detecting the presence of ships, obstacles, or land.

Secondary systems, such as shoran, hiran, Electronic Position Indicator, are useful tools to a ship engaged in hydrographic surveying where the location of the ship is desired relative to land areas in plotting soundings, maritime hazards or obstructions, etc., on nautical charts.

The radio compass or direction-finder has long been in use for fixing a ship's position. By use of this instrument, a ship can get her bearings on each of two radio transmitters operating at known locations and, by intersection of straight lines of bearing on a chart, determine the ship's position. (This system suffers from the error in the direction of the arriving radio waves.)

Like radar, one of the World War II developments in electronic equipment for navigational purposes resulted in the loran (LOng RAnge Navigation) system. This system was developed to allow a ship to fix her position without radio transmission from the ship. Operating on 2 megacycles, the loran system furnishes a ship at

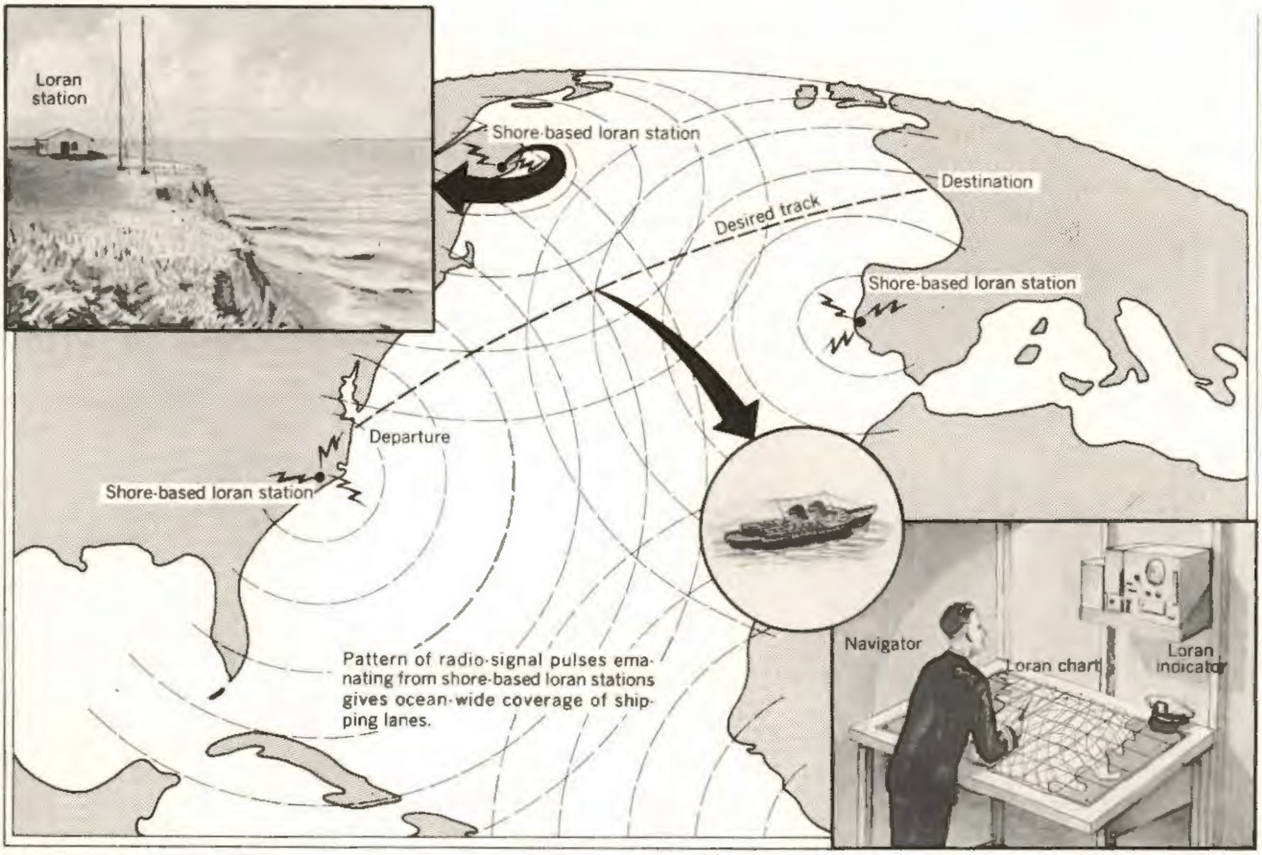

FIG. 3.--The Loran Navigation System

Departure and destination are known in earth coordinates (latitude and longitude). Desired track is established from knowledge of departure and destination in earth coordinates.

Actual position of ship can be determined by means of loran. Loran is a hyperbolic navigation system in which the navigating craft receives synchronized pulse signals from at least three known points. Time difference between arrival of signals from any pair of these transmitting stations is measured and determines a hyperbola-shaped line of position on a loran chart. By means of another pair of stations a second set of signals can be used to determine a second line of position. The crossing of the two lines of position gives a position fix in loran coordinates.

Loran chart shows loran lines of position superimposed on geographic plot. Therefore, location of present position in loran coordinates gives directly the location in earth coordinates.

sea two signal pulses, simultaneously emitted from two interconnected coastal loran stations 200 to 300 miles apart. These pulses actuate a loran receiver and indicator. The difference in arrival time of the two pulses aboard ship, in microseconds, can be ascertained, and the ship's position fixed on one of the hyperbolic lines of position shown on available loran charts. A second line similarly obtained from another pair of loran stations will determine the fix of the ship's position at their intersection.

In practice the two pairs of stations are created out of one master and two slave stations. The master station triggers the slave stations' pulses from its own transmission. The ship is able to distinguish between the two slave stations. Each charted loran line of position, which is a locus of equal time difference between pulses received at sea from the master station and a slave station, takes into account the radio transit time between master and slave stations and the time required to trigger the slave exciter. Since time differences alone are used in obtaining fixes, the system does not contain errors caused by directional arrival paths of the radio signals aboard ship. Accuracy of loran is acceptable up to about 700 miles in the daytime, 1,400 miles at night, but it is subject to ionospheric disturbances. ***The system is being adapted to low frequencies to extend the distance coverage.*** Figure 3 illustrates the use of the loran system.[2]

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[2]There are many other hyperbolic navigational systems as Gee, Decca, Lorac, Hyperbolic Raydist, Consol, etc., which are discussed in several sources, such as American Practical Navigator, Bowditch, 1958 ed., Ch.

Recent research (Sept. 1958 to Mar. 1959) by the Stanford Research Institute for the Department of the Air Force from stations at Fairbanks, Alaska, Thule, Greenland, and St. Johns, Newfoundland revealed the following information, assuming VLF (very low frequency) transmission:

1. Atmospheric noise is a function of receiver location and lightning location. Receivers farther north were farther from lightning activity (lightning is limited to middle latitudes and tends to follow the sun), and there was less atmospheric noise.

2. Drastic VLF propagation attenuation anomalies exist, which seem to be related to the earth's magnetic field. Also apparently related to the earth's magnetic field is the observed contamination of VLF propagation over long distances by all forms of polarization elements.

The implications are that designers of VLF navigation systems may have to abandon certain assumptions concerning radio propagation, such as the velocity of propagation is known and that the propagation path is a great circle.

ACOUSTIC SYSTEMS

As important to a ship as primary radar, particularly in shoal areas or offshore surveying, are sonar (SOund Navigation And Ranging) devices, both vertical (echo sounders) and directional underwater distance measuring devices.[3] The principle is basically the same as in primary radar. Short bursts of sound (usually in the ultrasonic range above audible frequencies) issued from the ship radially are reflected when they strike solid objects. The bearing and distance of such objects may be determined since the speed of sound in water is known and the time of travel of the sound waves to the object is measured. Such devices indicate range and bearing directly on a cathode ray tube as in primary radar.

The speed of sound in water generally decreases with depth until a minimum is reached, below which it then increases. The existence of this minimum speed level permits transmission of sound over distances in excess of 3,000 miles. If sound (as that of an underwater explosion) is created near the minimum speed level, and microphones are located at the correct depth, a single signal may be received at several widely spaced listening stations. As in loran, the differences in time of reception at these stations define hyperbolas which can be plotted on charts similar to loran charts, and hence the point of origin of the explosion can be determined. In a sense this is analogous to the loran system in reverse. It is called Sofar (SOund Fixing And Ranging). Rafos ("sofar" spelled backwards) is the reverse of sofar, sound signals being produced at the shore stations and the differences in reception times being determined at the ship, using a microphone lowered to the correct depth. This is the analog of loran for sound waves. (These systems are under development by the Navy Department.)[4] The Sofar effect was first discovered by scientists of the Coast and Geodetic Survey.

INERTIAL NAVIGATION SYSTEMS

The inertial navigation system depends essentially on four basic components: the accelerometer, the integrator (computer), the gyroscopic stabilized platform, and the platform servo.[5] Its name (inertial) stems from the fact that the large angular momentum of the fast spinning gyrorotor gives the gyroscope its basic property of maintaining a direction in inertial space. The direction of the spin axis of the gyroscope may be initially aligned with the local vertical at the starting point or parallel to the meridian through the initial point, etc., and it will maintain this under disturbances although precession of the gimbal in which it is mounted will occur.

The purpose of the gyroscopic stabilized platform and servo system is to keep the accelerometer level, since the accelerometer when not horizontal will detect a component of gravity and in the integration of the accelerometer output to obtain velocity (one integration) or distance (two integrations), errors will be introduced. Figure 4 depicts the basic system schematically.

This elementary system is basically the one used for ballistic missiles and satellite launching systems. A launch point reference is placed in the rocket by aligning a stable platform within the rocket to the local vertical and local north. Satellite injection data (or target data) are also inserted in terms of vertical and position. A system of precision accelerometers mounted on the stable platform sense deviations from the precomputed flight path and issue commands (through integrators) such as steering commands, propulsion orders, etc.[6] to control the rocket flight.

More sophisticated systems, pure and hybrid, are in existence and under development. Any moving vehicle on land, in air, on or under water,

XI; Radio Aids to Maritime Navigation and Hydrography, Special Publication No. 39, *International Hydrographic Bureau*, Monaco, July 1956.

[3]See Designing transducers for sonar systems, G. Rand, *Electronics*, Feb. 26, 1960.

[4]American Practical Navigator, Bowditch, *U. S. Navy Hydrographic Office*, 1958 ed.

[5]The ABC's of inertial navigation, E. Levinson, *(Sperry Gyroscope Special Report)*.

[6]*Ibid.*

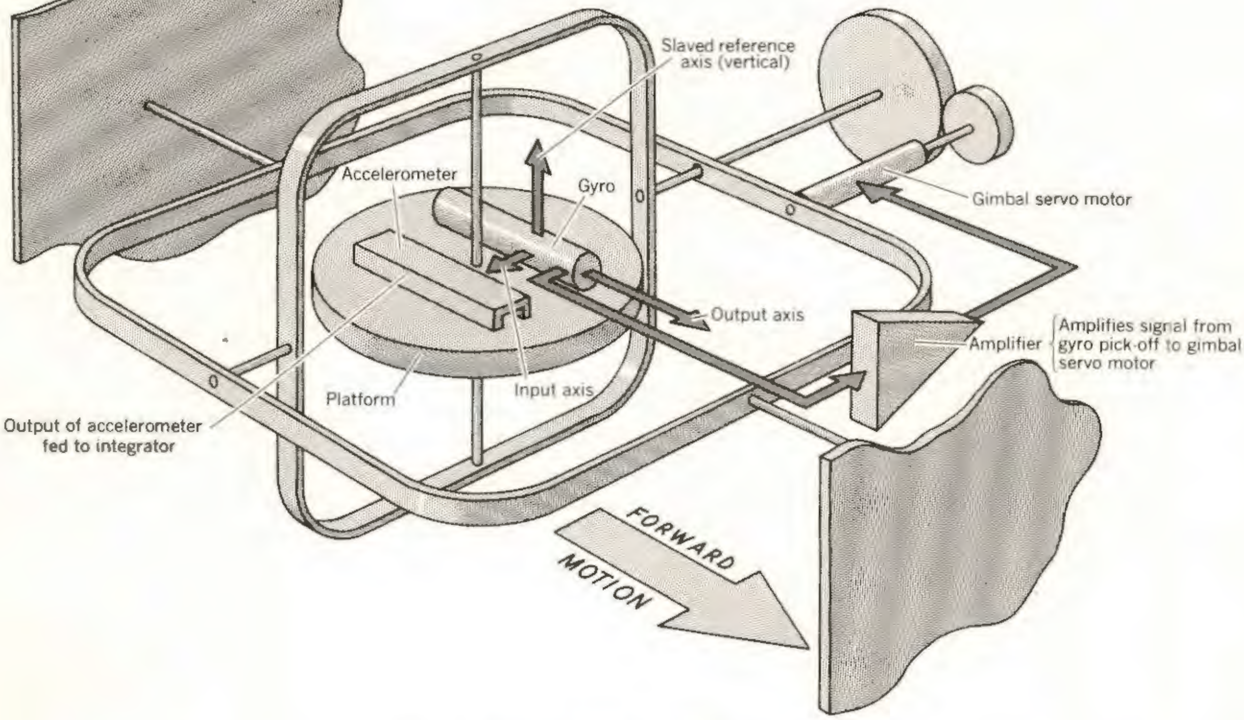

FIG. 4.--Simple Inertial Navigation System

Any motion about the input axis of the gyro (due to tilting of the vehicle and friction in the gimbal bearing) causes the gyro to precess about its output axis. A pickoff measures this precessional motion and sends an actuating signal to the gimbal servo motor. The motor drives the platform relative to the vehicle in a direction to cancel the original precessional motion. The action of the gimbal servo is thus to slave the platform to the reference axis (vertical) of the gyroscope.

or in space might have use for an inertial system. For instance an application for ship navigation results if one of the support axes of the gyroscopic package is slaved to be parallel with earth's axis of rotation. It is then possible to add a simple sidereal clock drive to compensate for the rotation of the earth as shown in figure 5a. The orientation of the reference member remains fixed with respect to inertial space about this polar axis, so that the first support gimbal outside of the sidereal time drive remains parallel to some meridian on the earth. By mechanical adjustments this artificial meridian may be aligned with any selected earth meridian. The combination of this artificial meridian inside the navigation equipment and the artificially established direction of the earth's polar axis provides an adjustable earth reference space that is derived from an adjustable inertial reference space and a time drive. Positions on the earth are fixed by determining the direction of the local gravitational vector with respect to this mechanically established earth reference space (see fig. 5b).

Thus pure inertial navigational systems are clearly dead reckoning systems (of a high order) and suffer from errors due chiefly to the drift of the gyroscopes used. Recent research and experiments with cryogenic gyroscopes indicate that mechanical bearings can be eliminated and the random drift rate reduced.[7]

An interesting recent hybrid development combines the ship inertial navigation system and a radio sextant, such systems having been designed for surface vessels and submarines. The radio sextant is essentially a radio telescope with a stabilized platform which will lock in on the X-band radio emission from the moon, the sun, or ultimately other stars besides the sun, and will provide corrections for the errors due to gyro

[7]Cryogenic gyro cuts random drift rate, P. J. Klass, *Aviation Week*, Feb. 1, 1960; see also Inertial guidance limitations imposed by fluctuation phenomena in gyro scopes, G. C. Newton, *Proceedings of the IRE*, April 1960.

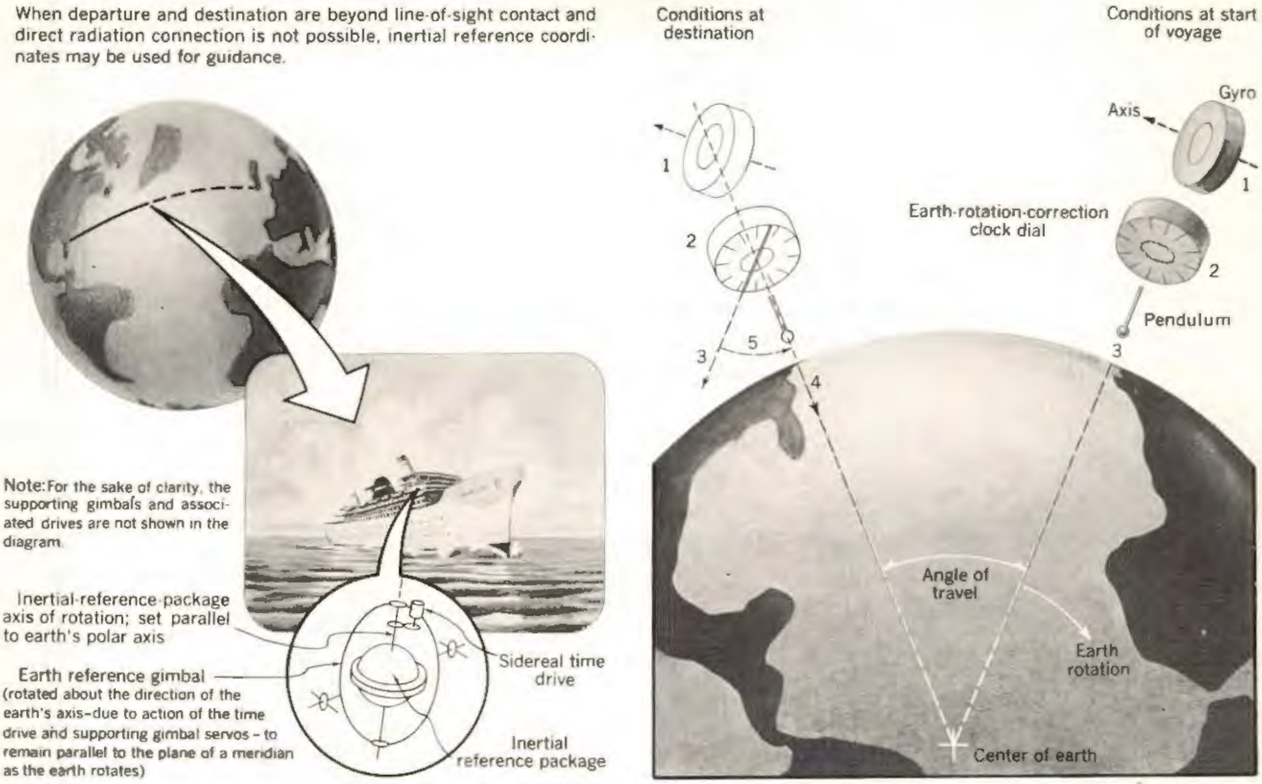

FIG. 5.—Application of inertial navigation system to ships.

(a)

Inertial-reference-package orientation is initially accurately established with respect to inertial space, that is, with respect to celestial space (for the purposes of practical guidance, inertial space and celestial space are effectively identical). This may involve the use of devices not included in the inertial guidance equipment.

High-performance inertial-reference gyro units (gyro units with low drift rates) supplying correction signals for servo-driven gimbals operate to hold accurately the inertial-reference-package orientation with respect to inertial space during the period guidance is required.

A specific force receiver system with Schuler tuning may be used to indicate accurately the vertical in moving ships. (See Appendix 1 for a definition of the "Schuler Pendulum.")

Measurement of the angles between the vertical and a member fixed with the proper orientation to the earth reference gimbal gives present position of the ship on the earth.

(b)

1. Gyro holds axis direction constant with respect to fixed stars, that is, with respect to an inertial reference space.
2. Dial rotation with sidereal time corrects for earth rotation to give an earth reference space.
3. Direction of gravity at start of voyage, with respect to the earth reference space.
4. Direction of gravity at destination represented by dotted lines, with respect to the earth reference space.
5. Indicated angle of travel is provided by angular displacement, in earth reference space, of direction of gravity with respect to its position at the start of the voyage.

drift in the ship's inertial system. Thus, an accurate all-weather navigation capability results.[8]

SUMMARY OF NAVIGATION SYSTEMS

Except for variations in established systems such as attempts to use VLF in loran, hiran improvement over shoran, etc., it is probably true that by the middle of the 20th century, radio and radar methods had practically eliminated the difficulties that attended the use of visual contacts for navigation by terrestrial landmarks. Equipment developments and methods had substantially exhausted the possibilities of improving navigation by terrestrial and celestial references. But loran is good only over ranges of 700 miles by day and 1,400 miles by night. There is no world coverage by loran and the system is affected by ionospheric disturbances. There is real need for an all-weather accurate positioning system, independent of ionospheric interference, in oceanographic surveys at greater distances from land, and all-weather navigation capability has always been desired for oceangoing vessels.

Military requirements for guidance systems in ballistic missiles, for positioning a missile-

[8]*Aviation Week*, Feb. 29, 1960, page 81; *Proceedings of the IRE*, Apr. 1960, page 500.

launching vessel at sea, for navigating submarines long distances under the sea, for guiding the rocket satellite carrier to the correct injection orientation to assure a specified orbit, etc., have resulted in development of inertial navigational systems and hybrid systems, some of which will provide navigation systems for lunar craft and interplanetary vehicles.[9]

The following table summarizes the system types in existence or under development.

ARTIFICIAL SATELLITES AND TERRESTRIAL NAVIGATION

In order to use an artificial satellite for terrestrial navigation, it is clear that an accurate ephemeris of its orbital motion is required, particularly the accurate correlation with respect to time of the geographic coordinates of its subsatellite points. Since this is also a requirement for a geodetic satellite, ***the satellite programs for geodetic and navigational satellites should be integrated to assure maximum utilization of any successful satellite launchings for either purpose.*** Other important considerations are the type of orbit most desirable, the number of satellites required for complete earth coverage, and the time available (frequency) for observations on a particular satellite and in the system of satellites.

Table 1.—Navigation Systems

System Type	*Radiation Medium*	*Examples*
Externally transmitted guidance	Receives external manmade radiation from radio stations, sound wave stations, etc.	Loran, radar "beam rider," rafos, etc.
Active	Radiated energy from ship and reflection gives intelligence	Radar (primary or search, Doppler homing), sonar, radio altimeter, etc.
Passive (radiation)	External sources of light, reflected light or radiation (light, heat, electromagnetic, acoustic)	Celestial navigation, optical sight, infrared heat tracker; sound, light, or radio seekers.
Passive (Non-radiation)	None	Magnetic compass, barometric devices, pure inertial guidance.
Hybrid	Combinations of above	Doppler-inertial,* stellar-inertial, radio command inertial, barometric-search radar, inertial-electronic sextant, etc.

*The Doppler principle is explained below.

The most desirable orbit for earth navigational purposes is circular because the height of the satellite above the earth (the distance from the satellite to its subsatellite point) will be nearly constant, the computation of ephemerides will be less complicated, and the satellite tangential velocity will be nearly constant. Accuracy at injection is very critical to achieve a circular orbit. In figure 11, if S is the point of burnout of the final stage of the rocket carrier, then to achieve a circular orbit the velocity vector V must be horizontal at the point S, within a few tenths of a degree, and the required injection velocity at altitude h cannot be in error by more than one-tenth of one percent. For instance, if h is 1,000 miles, figure 7 shows that V must be 15,820 mph and this cannot be in error by more than about 20 mph. Techniques are being developed by means of which an elliptical orbit can be made circular by installing a propulsion engine in the satellite so that it may be given a "kick in the apogee" and the perigee raised.[10]

In order to determine the most desirable height for a circular orbit, the earth coverage of a particular satellite at that height must be considered. Also, the number and inclinations for a particular height or heights, of satellites required to give complete earth coverage with reasonable time intervals of observations. The

[9]See: Navigation—From canoes to space ships, C. S. Draper, *Proceedings of the American Philosophical Society*, Apr. 1960; and Telebit—An integrated space navigation and communication system, G. E. Mueller, *Astronautics*, May 1960.

[10]Hearings before the NASA authorization subcommittee of the Senate Committee on Aeronautical and Space Sciences, Apr. 7-10, 1959, page 391.

precession of the orbital plane about the polar axis also affects the area of coverage of the satellite. Other factors to be considered are the amount of rotation of the earth during an orbital period, the width of the visibility belt, and the width of the belt in which the azimuth changes by more than 90°.

Figures 6 and 7, which were prepared by use of formulas (9) and (10) of Appendix 1, show the period T and tangential velocity V respectively as a function of altitude h, assuming circular orbits. Figure 8, prepared from formulas (3) and (5) of Appendix 2 gives the percentage of the area of the earth from which a satellite at a given altitude h is visible both from a single point of its trajectory and from one complete transit of its circular orbit. Figure 9 shows the width of the visibility belt (using formula (4) Appendix 2), the width of the belt in which the azimuth changes more than 90°, and the earth's rotation during one orbital period.

Figure 10 gives the rate of rotation of the orbital plane in degrees per day for inclinations of 15°, 45°, and 75° as a function of altitude h. Formula (7), Appendix 2, was used in preparing this figure. In Figure 11, the arc $A\ M_S A'$ is the arc width of visibility from the satellite S at altitude h, with subsatellite point M_S. The total visibility from the point S is the area swept out by the arc $A\ M_S\ A'$ as it revolves about M_S. The total visibility for one transit of the satellite in its orbit is the zone swept out by the arc $A\ M_S\ A'$ as the satellite traverses its orbit, the points A and A' describing small circles on the earth as shown. The inclination i is the angle which the orbital plane makes with the earth's equator.

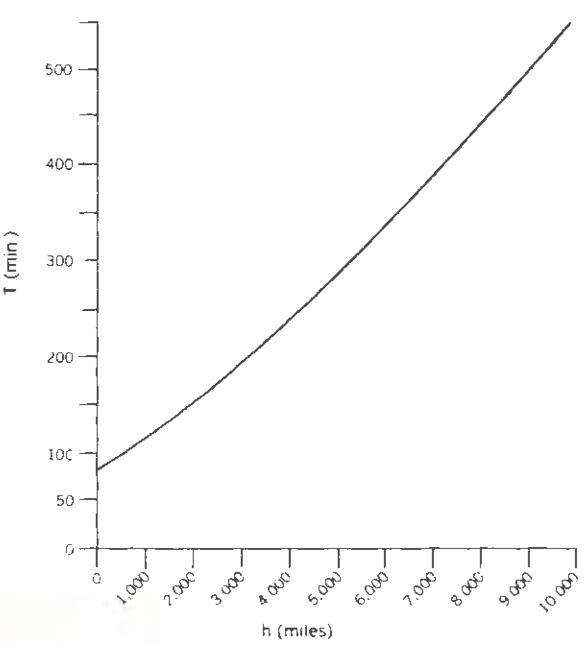

FIG. 6.—Circular orbits.
Period T as a function of altitude h.

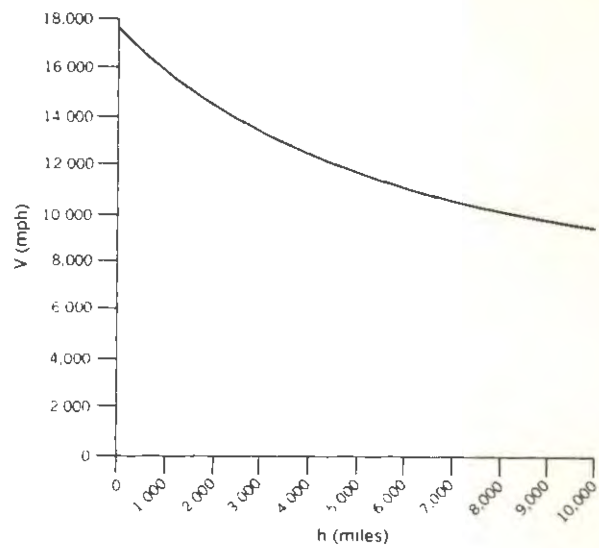

FIG. 7.—Circular orbits.
Tangential velocity V as a function of altitude h.

At the present state of development it is assumed that electronic radiation from the satellite will be the medium of locating the satellite from navigating vehicles on the earth—that is, ordinary celestial navigation instruments such as optical sextants cannot be used.

DISCUSSION OF CIRCULAR ORBITS AND SATELLITE REQUIREMENTS

From satellite orbit information amassed since the launching of Vanguard I, it is known that the air is denser at high altitudes than the presatellite empirical estimates indicated. Hence in order for orbits to be reasonably free from air resistance perturbations, the height h should probably be no less than about 500 miles. The effects of the earth's asymmetric gravity field on the orbit decrease with altitude (the chief effect is to cause the plane of the orbit to rotate about the earth's rotational axis—see Fig. 10). But at great altitudes the influence of the moon's attraction will complicate computation of ephemerides.[11] Thus, an optimum altitude h probably exists for minimizing the two chief orbit perturbing forces.

Recent studies of solar radiation pressure reveal that for a satellite of high area to mass ratio the height h can be reduced (from this force) 1 to 2 km. per day, reducing the satellite lifetime

[11]Secular perturbations due to the sun and moon of the orbit of an artificial earth satellite, C. C. Dearman, ABMA, Redstone Arsenal, Ala., ***Report RDSP-TR-5-59***, Nov. 1959.

considerably.[12] Since the satellite will carry transmitters (probably powered by solar batteries), the Van Allen radiation belts about the earth may influence the height h for navigational satellites. Recent studies of radiation damage to solar cells have given indications that for 2,000-mile satellite orbits the bare solar cells would be down to about 25 percent efficiency after 50 days. The findings also indicate that glass shielding 1/16 inch thick prevents irradiation of as high as 800 kev (kilo electron volts).

[12]The influence of the solar radiation pressure on the motion of an artificial satellite, P. Musen, *Journal of Geophysical Research*, May 1960.

Now from figures 6 and 7 the period T increases gradually with altitude, and tangential velocity V decreases rather rapidly with altitude to about 8,000 miles. From figure 8, the percentages of the earth as seen from a single point on the orbit and from a single orbit transit increase significantly with altitude up to about 6,000 miles. Table 2, compiled from figures 6-10, illustrates the chief parameters and their variation with altitude to 10,000 miles.

SPHEREOGRAPHICAL NAVIGATION BY MEANS OF ARTIFICIAL EARTH SATELLITES

Sphereographical navigation by means of satellites is not essentially different from ordinary

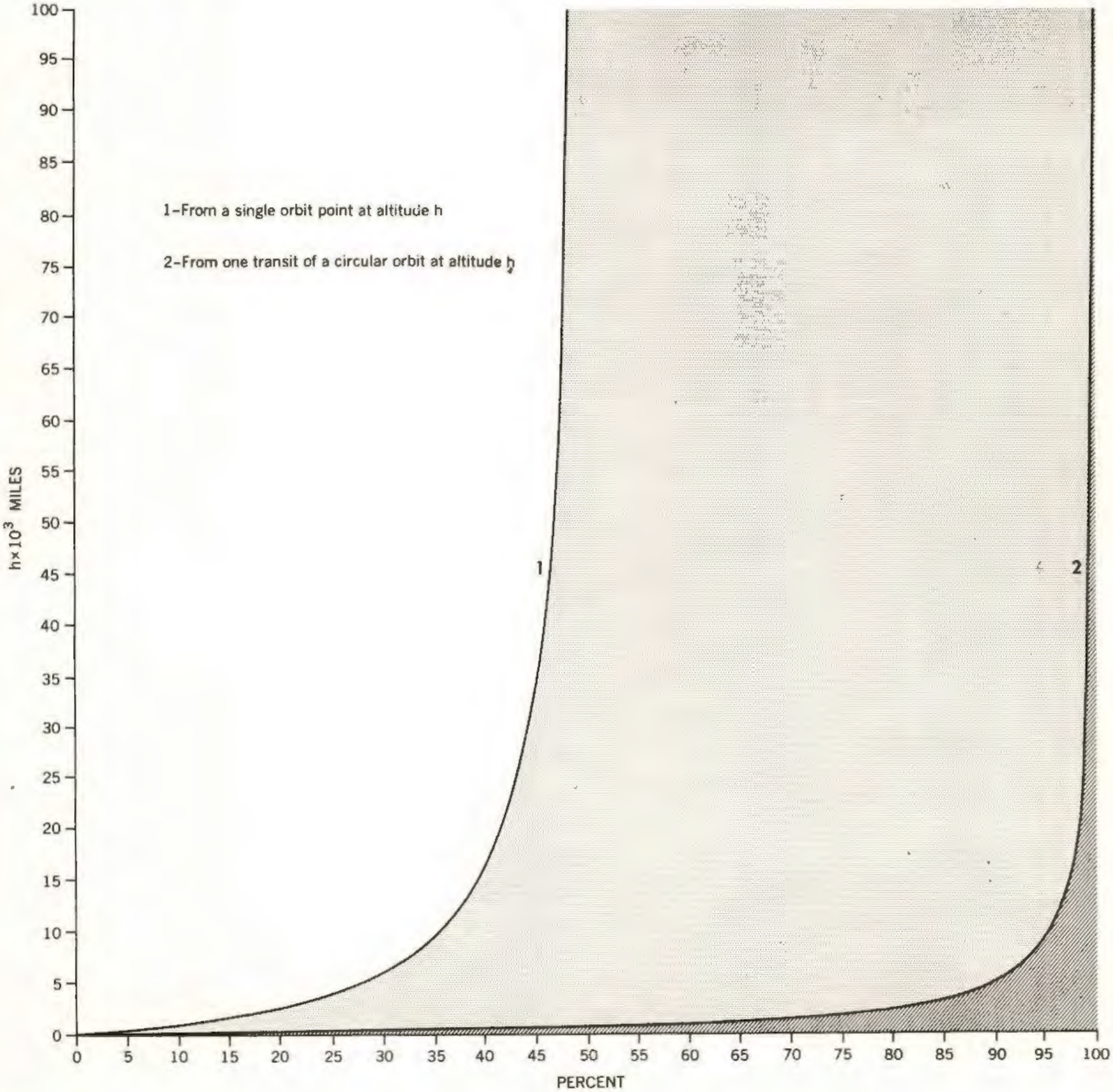

FIG. 8.--Percentage of the earth visible from an altitude h.

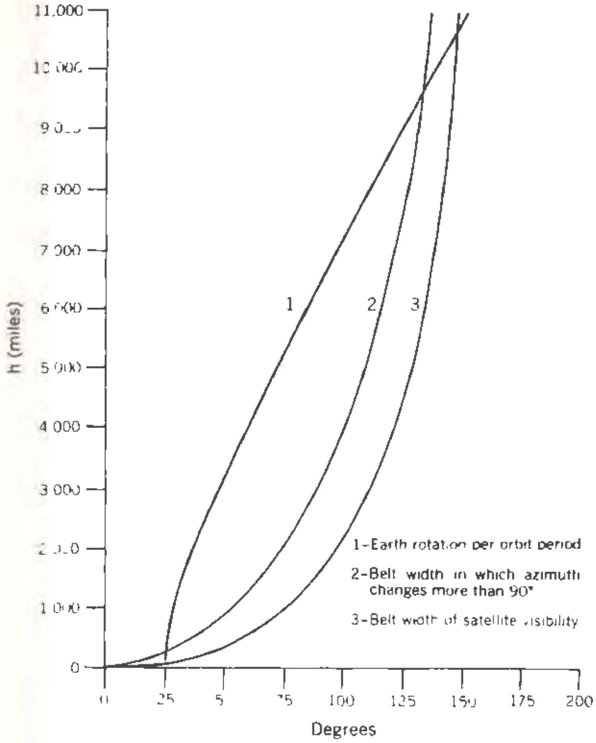

FIG. 9.–Circular Orbits.

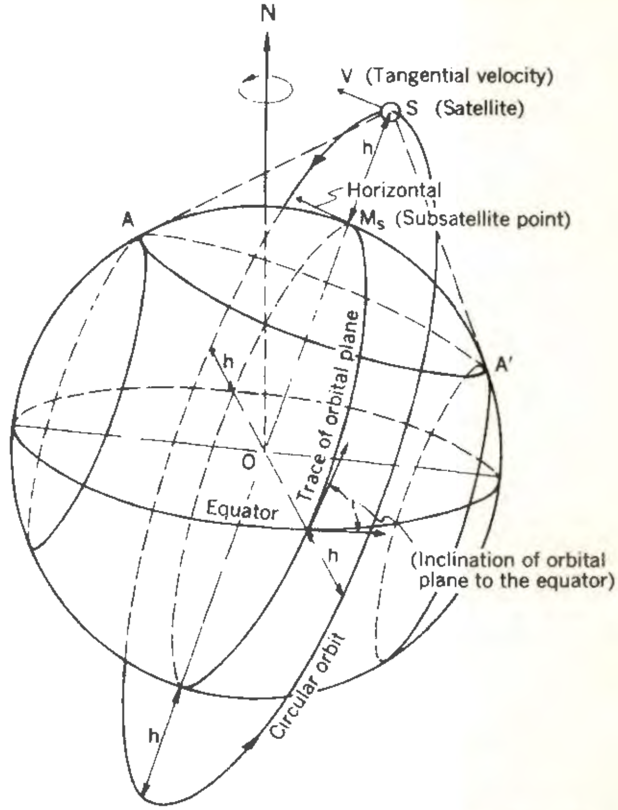

FIG. 11.–The arc $A\ M_S\ A'$ is the arc of visibility from the point S of the orbit. The area of visibility from the point S is the spherical cap generated by rotating the arc $A\ M_S\ A'$ about $O\ M_S$ as axis. The area visible from a single transit of the orbit is the spherical zone generated by the arc $A\ M_S\ A'$ as the satellite traverses its orbit at altitude h.

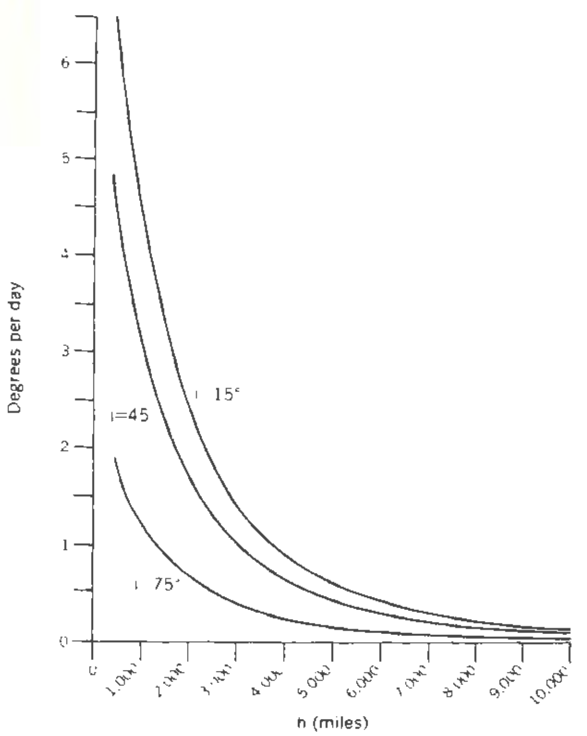

FIG. 10.–Circular Orbits.
Rate of rotation of orbital plane for inclinations 15°, 45°, 75° as a function of altitude h.

astro-position fixing. A device (pendulum or other) for indicating the vertical is needed and an accurate, highly directional, electronic sextant is required in addition to the usual chronometer. The satellite must radiate a continuous radio signal, which does not have to be accurately controlled in frequency, and an accurate ephemeris of the satellite motion must be available.

Development of an electronic sextant for use by surface ships has been sponsored by the Navy and the result is the radio sextant AN/SRN-4 (see fig. 12). Originally developed for use with radio radiation from the sun and moon, it could be adapted for use with satellite radiation in the microwave band after some developmental changes. The two major components of this instrument are (1) the tracking unit, consisting of the antenna and its scanning mechanism, the receiver, the tracking servos, and the angle readout system, (2) the Schuler-tuned stabilization system and north reference. (See Appendix 1 for a definition of the Schuler-pendulum.)

Table 2.—Circular Orbit Data

h	T	V	Earth rot. per T	Orbit plane rot. (i=15°)	Belt width of visibility	Area vis. from orbit point	Area vis. one transit
mi.	*min.*	*mph.*	*deg.*	*deg./day*	*deg.*	%	%
500	101	16,680	27	+6.35	55	5.6	46.0
1,000	118	15,820	30	+4.40	74	10.1	60.2
2,000	156	14,430	39	+2.30	97	16.8	74.7
3,000	197	13,350	48	+1.33	111	21.6	82.2
4,000	240	12,490	60	+0.84	120	25.1	86.7
5,000	287	11,770	70	+0.55	127	27.9	89.7
6,000	337	11,160	83	+0.38	133	30.1	91.7
7,000	389	10,640	95	+0.27	138	31.9	93.2
8,000	443	10,190	110	+0.20	141	33.4	94.3
9,000	500	9,790	124	+0.14	144	34.7	95.2
10,000	560	9,430	140	+0.12	147	35.8	95.9

FIG. 12.—Radio Sextant AN/SRN-4.

Since satellite orbits at an altitude of from 1,000 to 10,000 miles would be more predictable, fewer satellites would be needed for world coverage, and ephemerides could be prepared longer in advance, it would appear that as an extension of celestial navigation, this approach is feasible, particularly if the "running fix" type of navigation is used. Some additional advantages would be a continuous north reference in all weather and a virtually jam-proof passive system.[13]

[13]See Navigation using signals from high altitude satellites, A. B. Moody, *Proceedings of the IRE*, Apr. 1960; and A precision study of satellite orbits and their use as an aid to terrestrial navigation, Report CEP-1307, *Collins Radio Co.*, Cedar Rapids, Iowa, Sept. 26, 1958.

Some disadvantages could be the complexity, amount and cost of required equipment aboard each ship. (The physical dimensions of the antenna system necessary to achieve the required accuracy could be prohibitive.)

THE DOPPLER SHIFT TECHNIQUE

In 1842, Christian Johann Doppler, an Austrian physicist and mathematician, published a paper "Uber das farbige Licht der Dopplesterne" (On the colored light of the double stars), which contained what is now known as the Doppler effect. He drew the analogy between the sound coming from a moving source and the light coming from a moving star; as the pitch of sound varies (falling suddenly and very markedly in pitch as it passes close to the observer and recedes), so Doppler erroneously thought the color of the light from a star would be altered. In 1848, Armand Fizeau gave the correct explanation of Doppler's principle in the optical case showing that the effect would manifest itself as a slight shift of the bright or dark lines of the spectrum toward the violet if the source is moving toward the observer. The reverse is true for a source moving away. The principle is used in astronomy for the discovery of double stars. For other types of waves, the principle is analogous except that a shift of frequency is usually observed rather than a shift of spectral lines.

For illustration, suppose that a ship is at anchor in a seaway. If the waves are regular, a certain number of them will pass under the ship during a given interval of time. If the ship were moving directly into the waves, it would encounter more than that number during the same interval, and if it were moving with the waves it would encounter fewer. The frequency of the waves as observed from the ship would vary according to the ship's motion and so would their wave length. When the ship is at anchor, let f_0 be the number of waves of a certain definite wave length λ and velocity c passing under the ship in one second. That is, the ship will receive f_0 of these waves every second as long as the ship is anchored. Now suppose the ship is moving with the waves at a uniform velocity v. Then in a second of time, f_0 waves reach the ship's position just as when anchored but at the end of that one second of time the ship is at a distance v times one second beyond its initial position, and hence it is impossible that all of these f_0 waves can have reached it. The deficit will be equal to the number of waves whose combined lengths would just measure the distance v. But f_0 waves are sent out every second and when the last of these is just leaving its source, the first one has gone a distance equal to c times one second or c. Now the length of one wave length is $\lambda = c/f_0$, and to find the number of wave lengths that would be required to fill the distance v, we have only to divide v by the length of a single wave, or $v/\lambda = f_0 v/c$. This is the amount that f_0, the number of wave lengths being received per second (the frequency), must be diminished because of the ship's motion with the waves. That is, when the ship is running with waves at a velocity v, it will receive only $f_0 - f_0 v/c$ waves per second, and the wave length will appear to be longer than it really is. If the ship is moving directly into the waves at a uniform velocity v, it will receive $f_0 + f_0 v/c$ waves per second and the wave length will appear to be shorter than it really is.

Now consider a satellite transmitting a continuous unmodulated radio signal at a fixed frequency, f_0, as it approaches and passes the ship. If s is the distance from the ship to the satellite, then $v = \dot{s} = ds/dt$ (derivative of s with respect to time) is the radial velocity of the satellite, and the change in frequency caused by the Doppler effect is v/λ where $\lambda = c/f_0$. But for radio waves c is the velocity of light and as the satellite approaches the ship the frequency being received will be $f = f_0 + f_0 v/c$. Now from figure 13 it is seen that when the distance s is at a minimum, s_m, $v = \dot{s} = 0$, and at that instant the frequency being received at the ship is the true satellite transmitter frequency, f_0. Then as the satellite recedes, the sign of $v = \dot{s}$ is negative and the frequency being received at the ship is $f = f_0 - f_0 v/c$. Now, if a receiver and recording device operate during the transit of the satellite in the neighborhood of the ship, then from the resulting graph of frequency versus time, as shown in figure 14, the "shift" in frequency is clearly discernible and the time of closest approach to the orbit is detectable. Knowing this time of closest approach the ephemerides of the satellite motion will give the geographical coordinates of the subsatellite point M_s as shown in figure 13. Since the orbit is assumed circular, the altitude h and tangential velocity v are known. (If the orbit is not circular then these will be

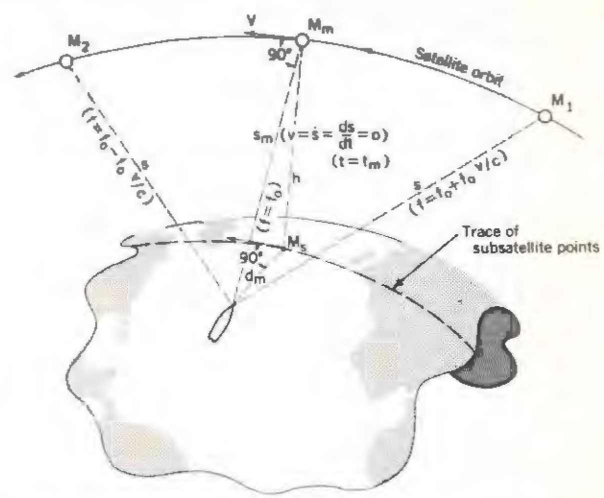

FIG. 13.—Illustrating the basic parameters and geometry involved in locating a ship at sea by the use of the Doppler frequency shift of the satellite radio transmission.

known from the ephemeris data.) Now s_m, the minimum approach distance, can be computed and also d_m, the distance from the ship to the subsatellite point M_s. If the trace of the subsatellite points is plotted on a conformal map (one in which angles are preserved) and the point M_s located on this trace; then the distance d_m laid off at right angles to the track at M_s gives the location of the ship (see app. 3). Usually a second such fix on a second successive transit of the same satellite, or from a second satellite (of known ephemeris) will be used as a check, particularly if the ship is near the subsatellite track on the earth.

While in theory the method is feasible, there are problems associated with establishing such a Doppler navigation system. One of these is the need of an extensive accurate tracking system to establish an adequate ephemeris. Predictions of the Vanguard I orbit a few hours in advance are off by less than one mile; less than 5 miles over a period of a few days, but are off by some tens of miles after a month.[14] Since the sources of perturbations of satellite orbits are being extensively studied and orbital theory is being improved constantly, it is believed that the prediction problem will not be a serious handicap. The several satellites necessary to assure world coverage must have orbits as nearly circular as possible and in orbital planes of correct inclination. While the inclination is controllable to a reasonable degree, the requirements for a circular orbit impose stringent injection requirements. But the improvements in rocket carriers and guidance systems, as evidenced by several

[14]Space handbook, astronautics and its applications. Staff Report: U. S Congress Select Comm. on Astronautics and Space Exploration, 1959, page 200.

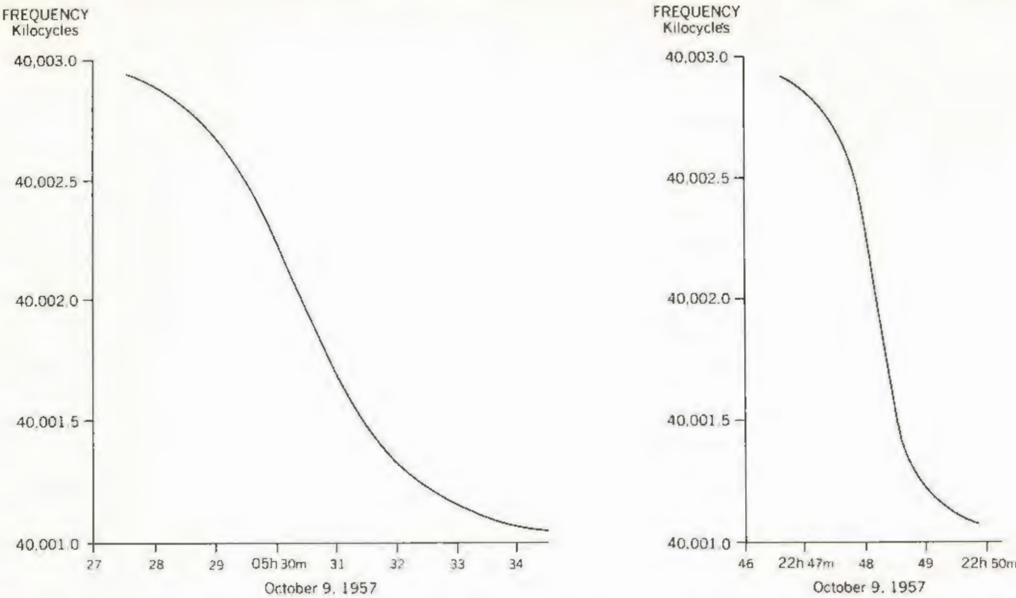

FIG. 14.--The apparent frequency of the radio transmitter in Sputnik I, passing at ranges of 350 and 135 nautical miles respectively.

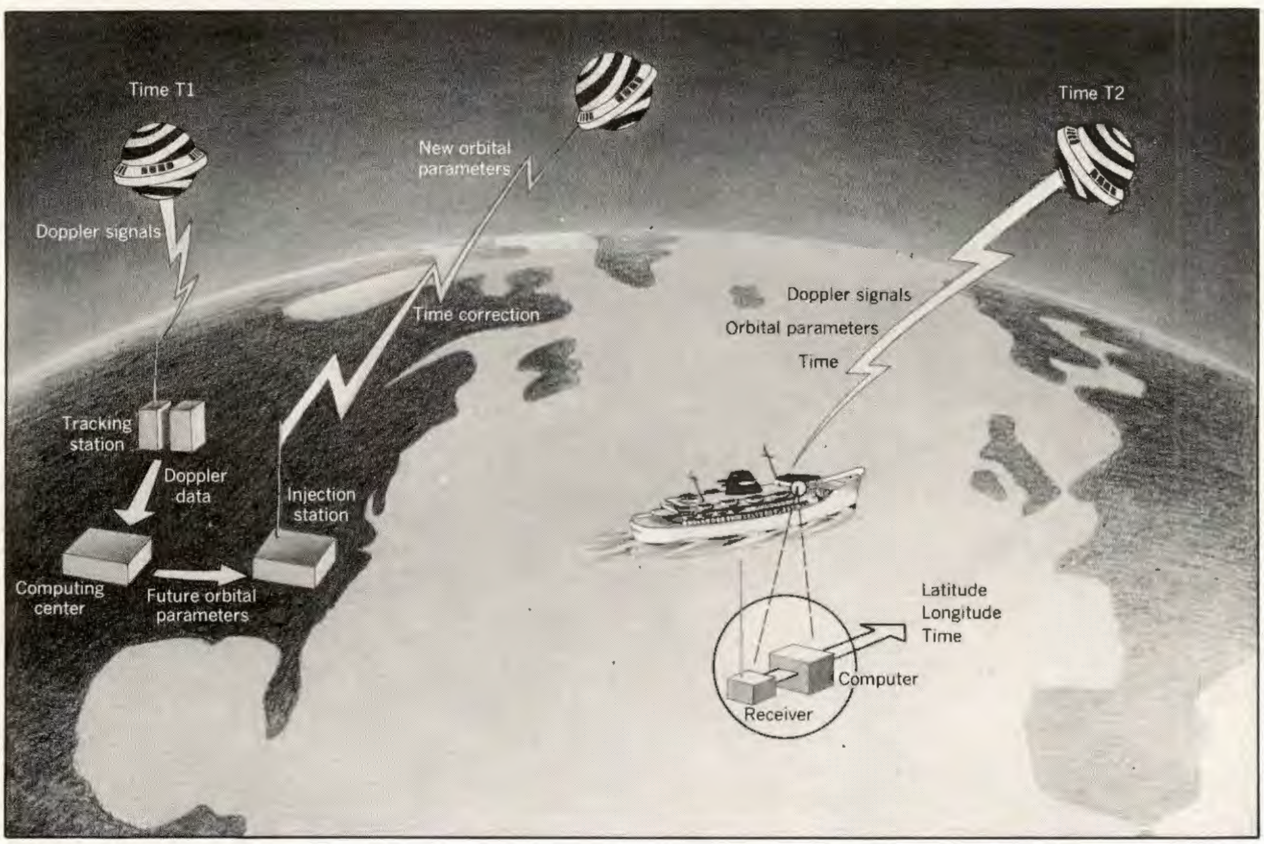

FIG. 15.--After launching of Transit I-B the satellite's Doppler signal is picked up by receiving station (time phase 1), which reduces it to digital form and transmits it to Central Computer Center at Applied Physics Laboratory of The Johns Hopkins University, Silver Spring, Md. Staff at computer center, using Doppler data, is able to predict future paths or orbital parameters of Transit I. These are relayed to injection station and then transmitted back to Transit I. Satellite stores the data and transmits it to earth intermittently (time phase 2). Ship navigators with receiver-computer equipment combine position information of satellite with current Doppler shift data to determine their position on earth.

recent satellite launchings, indicate this to be a transient handicap. Since the success of the Doppler technique depends on accurate detection of small shifts in radio frequencies transmitted by the satellite, ionospheric interference cannot be tolerated. The solution here seems to be both higher frequencies (line of sight transmission) and several simultaneous frequencies (harmonically related) transmitted from the satellite. Excessive rotation of the satellite must be damped, since the resulting modulations of the satellite transmission would affect the Doppler frequency shift.

THE TRANSIT NAVIGATIONAL SATELLITE SYSTEM

The Bureau of Weapons, Department of the Navy, through Advanced Research Projects Agency (ARPA), Department of Defense, and the Applied Physics Laboratory, The Johns Hopkins University, initiated a developmental program for a navigational satellite using the Doppler shift technique. This program is now completely the responsibility of the Navy. The program is actually aimed at two compatible simultaneous receiving systems, that is one transmitting system for the satellite and two receiving systems for navigating vehicles to allow navigation within a nautical mile with the less accurate receiving equipment, and positioning to within a tenth of a mile with the accurate receiving equipment.

Present plans call for the satellite to transmit not only a very stable constant unmodulated frequency signal for utilizing the Doppler principle but also to transmit its instantaneous orbital position (at intervals of one minute). That is, the parameters from which the orbital data needed at the time of observation can be computed. Also to be included in the final version is an accurate clock (the satellite will transmit universal time). To make this possible the Navy will maintain, initially and during the research and development phases, a chain of tracking stations around the earth which feed their data into a central computing center, which in turn will transmit to the satellite the orbital correction data for retransmission to navigating vehicles (see fig. 15). The final operational navigation system will not require tracking stations around the earth but only on the North American Continent. The satellite will always transmit the most accurate data available to the greatest number of meaningful significant figures. At present the Navy has 7 tracking stations in operation; these are located at the Applied Physics Laboratory, Silver Spring, Md.; University of Texas; University of Washington; University of New Mexico; Argentia, Newfoundland; the Royal Aircraft Establishment, Lasham Hants, England; and San Jose dos Campos, Brazil.[15]

The precise receiving equipment will use the maximum accuracy of the parameters as transmitted by the satellite. In addition to the very sensitive receiver, frequency control, etc., an electronic high speed computer will be required aboard ship. The less accurate equipment (which will not use all the inherent accuracy of the satellite transmission) will not require an electronic high speed computer; only a slide rule or desk computer will be needed.

The development is being conducted in four phases. The first phase is the feasibility study. Transit I-B, see figure 16, which was placed in orbit on April 13, 1960, is one of this series. Apogee at launch was about 750 miles, perigee 230 miles, inclination about 51°, and period about 96 minutes. It carries transmitters of four frequencies—54, 162, 216, and 324 megacycles—to study ionospheric effects on transmission. It also carries an infrared scanner (see fig. 16) as developed by the Naval Ordnance Test Station to measure the earth's albedo (reflecting power) for infrared radiation. Instrumentation for other scientific experiments is also being carried.[16]

Transit II-A, placed in orbit on June 22, 1960, is similar to Transit I-B, figure 16. It weighs 223 pounds; its orbit has a perigee of about 400 miles, apogee of 660 miles, period of 102 minutes, and inclination of 67°. Shortly after it had achieved orbit, a spring mechanism released a smaller 42-pound satellite, developed by the Naval Research Laboratory for measuring solar radiation, which has an 108 megacycle transmitter for telemetering this information back to earth (see fig. 17). Transit II-A, in addition to the transmitters as carried by Transit I-B, is carrying for Canada a receiver to study background radio noises from the galaxies. It is also carrying an electronic or digital clock.

The second stage of the development will be for evaluation of engineering design data; the third stage will be for prototype operational satellites; the fourth and final stage will be the operational transit satellites.

In the final system, it is hoped to have four fully instrumented satellites, two at inclinations of 67°5, two at 22°5, with the orbital planes of each pair 180° apart.[17] The planned orbits are circular at an altitude of 500 miles to minimize air resistance and gravitational effects and to keep

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[15]*Aviation Week*, Apr. 18, 1960, pages 29-31; see also *U.S.C. & G.S. Tech. Bull. No. 11*, page 33.

[16]*Aviation Week*, Mar. 28, 1960, page 26.

[17]*Astronautics*, June 1960, page 104.

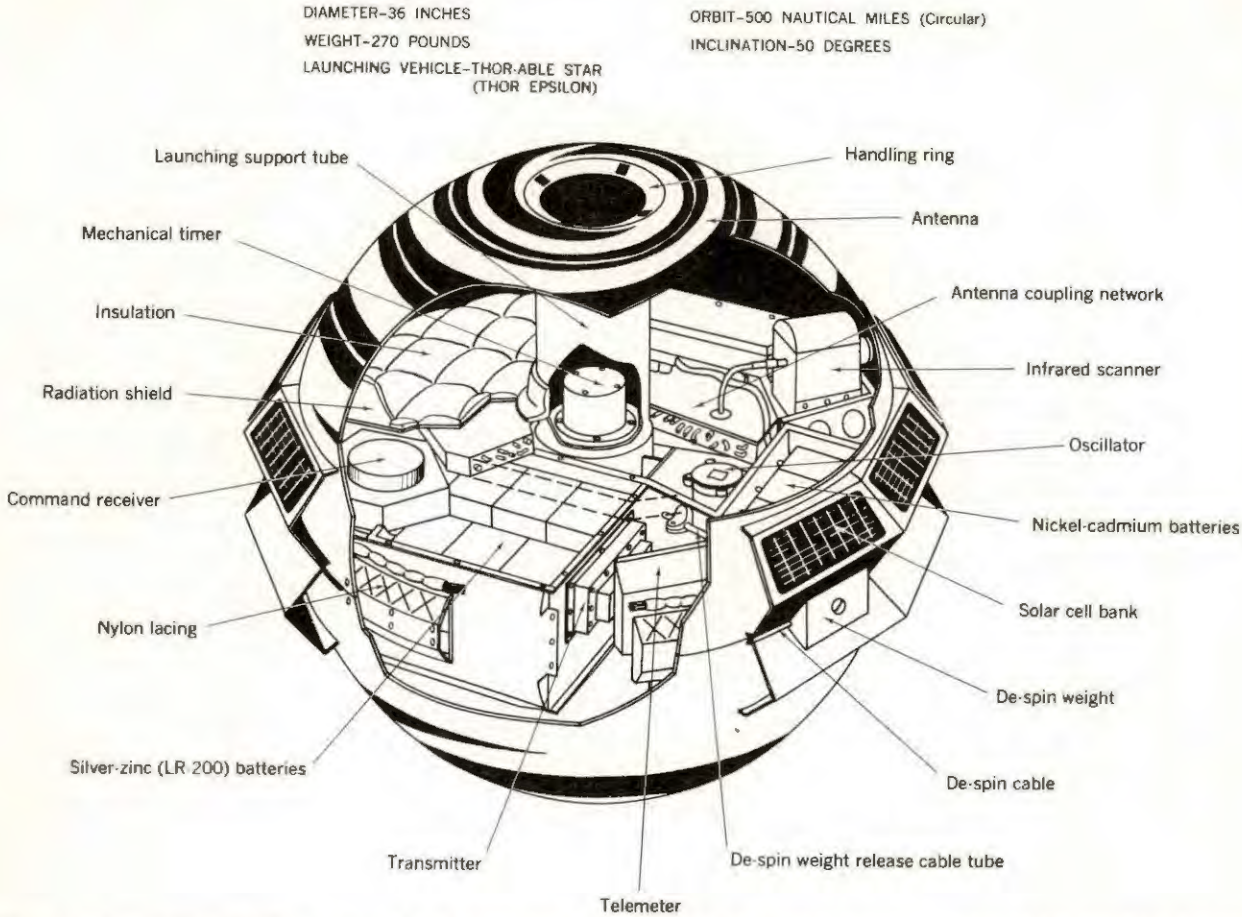

FIG. 16.--Cutaway drawing of Transit I-B shows how equipment is shelved in the midsection of the satellite for optimum balance. Despin systems will introduce counter-force to retard the satellite's initial spin.

the satellites below the Van Allen radiation belts. If these orbits are attained, an observation should be possible every 90 minutes anywhere on the surface of the earth. To give more complete coverage two additional satellites may be added, one in polar orbit, the other in equatorial orbit.

The final transit satellite will be smaller in size and weigh less. By transistorizing the instrumentation and eliminating all superfluous equipment the final version should weigh only 50 to 100 pounds, making possible the use of a less expensive rocket injection system, such as the NASA Scout now under development.[18] The transmitters of the operational satellite should have a lifetime of about 5 years, making necessary the replacement of about one satellite per year to maintain the system.

The initial spin of the research and development transit satellites will be retarded almost immediately after orbit is attained by two weights located diametrically opposite each other at the sphere's midsection, see figure 16. These weights, their attaching cables wound round the sphere in a direction opposite that of the satellite's induced spin, will be automatically ejected by a preset decelerating timing device. Upon unwinding, the weights will introduce a counterforce to retard the satellite's rotation (and thus prevent modulation of the satellite transmission). In the operational satellites, despin will probably not be necessary, but if it is, a magnetic despin device will probably be used.

Since coefficients in the mathematical expression for the earth's gravity potential and hence geodetic parameters as the flattening may be deduced from accurate satellite orbital data,[19] it is seen that the orbits of the transit satellites may be used for this purpose. This is particularly important since to study adequately the earth's asymmetric gravity field, several satellites at different inclinations are necessary. Hence, this should be a most productive byproduct of the

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[18] *Aviation Week*, Apr. 18, 1960, pages 29-31.

[19] See *U.S.C. & G.S. Tech. Bull. No. 11*, pages 25-30.

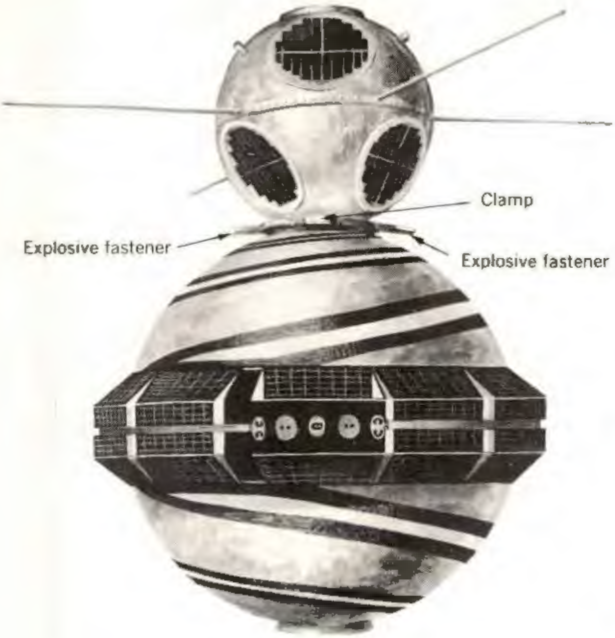

FIG. 17.—Solar cell energy sources on the Transit II-A satellite are located in a band around its circumference; on the smaller satellite installed above for radiation measurement, the cells are placed in circular ports. Antenna on Transit II-A is the white spiral band painted on its surface. The radiation measurement satellite uses whip-type antennas. The two satellites are held together by a single clamp. As the satellites were separated from the final rocket stage, the clamp was blown free by two explosive fasteners, and a compressed spring pushed them apart.

transit satellite program and of course allow, as orbits are better determined, advanced predictions which may be issued as ephemerides of the satellites' motions to be published far enough in advance to make possible utilization of less expensive equipment aboard ship.[20] One proposed transmitter is compact enough to be carried "hitch-hike" on most earth satellites (for instance it could have been carried by Tiros). The receiver is also compact (19" x 10 1/2" x 18"), weighs 50 pounds, and requires 75w. for operation. The cost of the shipboard equipment is estimated at $1,000.

One of the Transit satellites to be launched in late 1960 or early 1961 will carry a small 6.5 pound transponder—part of the secor (SEquential Collation Of Range) tracking system which has been developed. It is a distance measuring system.

Each of the secor ground stations determines the distance of the satellite by transmitting a frequency-modulated, continuous wave signal which is transmitted back by the satellite transponder at an offset carrier frequency, the phase shift between the transmitted and received signals as measured at the ground station being a measure of the distance from the ground station to the satellite.[21] During each passage of the satellite each secor ground station will make repeated distance measurements and an adjustment technique will be used to determine the distances on the ground to secor stations whose geographical positions are unknown. Obviously the geographical positions of some of the secor ground stations must be known (located with respect to established geodetic datums or by astronomical observations).

The Navy hopes to have four prototype satellites in orbit by 1962 and the operational transit satellites as soon thereafter as practicable.

We conclude with mention of a study of a navigational system using satellites by one company;[22] and of a system being developed, called MISTRAM (MISsile TRAjectory Measurement), which makes use of range and range-rate in tracking missiles and satellites, and which if successful should have application to both navigational and geodetic satellite programs.

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[20]NAVSAT, Worldwide satellite navigation system, BSD-217, *Bendix Aviation Corp.*, Systems Div., Ann Arbor, Mich., Jan. 1960.

[21]*Aviation Week*, Apr. 25, 1960, pages 28 and 29.

[22]Study of a navigation system utilizing satellites, Part I, Maritime and airline applications, Systems Development Laboratories, *Hughes Aircraft Co.*, Oct. 20, 1959.

APPENDIX 1

A SIMPLE TREATMENT OF THE RELATION BETWEEN SATELLITE VELOCITIES, PERIODS, AND HEIGHTS

Lansing G. Simmons

Newton's law of gravitation states that any two particles of matter attract each other with a force proportional to the product of their masses m_1, m_2 and inversely proportional to the square of the

$F = am$

distance d between them. This may be expressed as

$$F = G m_1 m_2 / d^2, \tag{1}$$

where G is the constant of gravitation.

This law, equation (1), can be applied to the earth and the moon, or the sun and the planets, with a good degree of approximation even though these bodies are not particles. For it turns out that, if the body is spherical and the density within it depends only on the distance from the center, the action on bodies outside the sphere is exactly the same as though all matter were concentrated at its center. We shall make these assumptions for all attracting bodies in the discussion that follows. We shall also assume circular orbits and terrestrial gravity unaffected by the centrifugal force of the rotating earth. These assumptions, and those omitting gravitational effects of other bodies, although inadmissible in precise calculations, are safe enough for the purpose at hand.

From elementary physics we learn that if a particle is traveling at a uniform velocity v_0 around a circle whose radius is r, its acceleration toward the center is

$$a = v_0^2 / r. \tag{2}$$

For a particle of mass m the centrifugal force F_c is mass times acceleration, whence from (2)

$$F_c = ma = mv_0^2 / r. \tag{3}$$

Now consider a small satellite in circular orbit around the earth and close to its surface. Neglecting air friction, and with the assumptions previously stated, there are two forces to balance, those in equations (1) and (3). Assuming M as the mass of the earth, m the mass of the satellite, and R the radius of the earth, we find from (1) that $F = GMm/R^2$. Since this is the weight of the satellite then GM/R^2 equals g_0 or surface gravity and $g_0 m$ is then the weight. This is counterbalanced by the centrifugal force from (3), so we have $g_0 m = mv_0^2/R$, and solving for v_0

$$V_0 = (g_0 R)^{1/2}. \tag{4}$$

This gives the velocity of a satellite at the surface of the earth in terms of the earth's radius and surface gravity.

We see from equation (1) that the gravitational force varies inversely as the square of the distance. At a distance d from the center of the earth we must replace g_0 with $g_0 (R/d)^2$ and the velocity v of a satellite at the distance d would be from equation (4)

$$v = (g_0 R^2 d/d^2)^{1/2} = (g_0 R^2/d)^{1/2}. \tag{5}$$

Dividing respective members of (5) by those of (4) we have

$$v/V_0 = (R/d)^{1/2}, \tag{6}$$

where for circular orbits of satellites $d = R + h$, h being the height of the circular orbit above the earth.

Now the period of a satellite (time of one revolution) varies inversely with the velocity and directly with the distance from the center of the earth. Letting t_0 be the period of a surface satellite and t the period of one at distance d, we derive from (6)

$$t/t_0 = (d/R)^{1/2} (d/R) = (d/R)^{3/2}, \tag{7}$$

or

$$t^2/t_0^2 = d^3/R^3. \tag{8}$$

Equation (8) is actually Kepler's so called "Harmonic Law" which states that the squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. In our case the earth replaces the sun and the satellites replace planets.

We can now place some numbers in these equations. In equation (4) we may assign for g_0 a value of 32.2 ft./sec.^{2} or 79,040 mi./hr.^{2} and for R a value of 3,960 miles. v_0 comes out as 17,692 mph. Now $t_0 = 2\pi R/v_0$ (hours), hence $t_0 = (6.2832)(3960)(60)/(17,692)$ min. or 84.4 minutes. Note that using equation (4), we have $t_0 = 2\pi R/v_0 = 2\pi R/(g_0 R)^{1/2} = 2\pi (R/g_0)^{1/2}$. Now this is the period of a simple pendulum of length R (the radius of the earth). It is known as the "Schuler Pendulum" and has the same period as that of a surface satellite, 84.4 minutes.

From (7)

$$\begin{aligned} t &= t_0 (d/R)^{3/2} = (84.4)(d/3960)^{3/2} \\ &= (84.4)(1 + h/3960)^{3/2}, \end{aligned} \tag{9}$$

and from (6)

$$v = (1.114)(10^6)/d^{1/2} = (1.114)(10^6)/(R+h)^{1/2} \tag{10}$$

where h is the altitude of the satellite above the earth.

Suppose h is 1,000 miles, what is the period? From (9) $t = (84.4)(1 + 10^3/3,960)^{3/2} = 118.3$ minutes. If the orbit is elliptical we may take the average between perigee and apogee as h with good results.

Let us apply this to the moon. The mean distance is about 238,000 miles. So we have $t = (84.4)(238,000/3,960)^{3/2} = 39,300$ min. $= 27.3$ days which is very nearly the sidereal month.

There is the interesting case of the so called "stationary" satellite. If such a satellite is launched eastward from a point on the equator, and given a certain velocity at a critical altitude it would remain over the same point on earth. The period, of course, is one day, or $t = 1{,}440$ minutes. What is the necessary altitude and velocity? From (9) we have $1{,}440 = (84.4)(1 + h/3{,}960)^{3/2}$ and solving for h

$h = (3{,}960)(1{,}440/84.4)^{2/3} - 3{,}960 = 26{,}250 - 3{,}960 = 22{,}290$ miles. The velocity is $2\pi d/t = (6.2832)(26{,}250)/(24) = 6{,}870$ mph. The same result is obtained from equation (10).

Thus we see that if the altitude, velocity, or period of a satellite is given, the other two may be easily computed to 3-figure accuracy. We need to keep in mind that the time and distance units must be consistent in using the equations.

APPENDIX 2

FORMULAS USED IN COMPILING GRAPHS

In figure 18, M_S is the subsatellite point, P is an arbitrary point on the earth (considered spherical), O is the center of the earth. The following formulas may be written from the geometry of the figure:

$$\sin(\theta/2) = (1/2)\left[(s^2 - h^2)/R(R+h)\right]^{1/2},$$
$$\cos(\theta/2) = k/2R$$
$$k^2 = R(s^2 - h^2)/(R+h) = h^2 + s^2 - 2hs\cos\phi$$
$$= 2R^2(1 - \cos\theta)$$
$$(R+h)/\sin(\phi+\theta) = R/\sin\phi = s/\sin\theta, \quad w = R\theta,$$
$$s = (R+h)\sin\theta\csc(\phi+\theta) = R\sin\theta, \csc\phi$$
$$\theta = \arcsin\left(\frac{h+R}{R}\sin\phi\right) - \phi. \qquad (1)$$

The area of the spherical cap about M_S of arc radius w is

$$A_Z = 2\pi R^2(1 - \cos\theta) = \pi k^2. \qquad (2)$$

Now when $\theta + \phi = 90°$, then $\cos\theta = R/(h+R)$, and the spherical cap about M_S representing the area from which the satellite may be seen at an altitude h is then from (2) with $\cos\theta = R/(R+h)$

$$A_Z = \pi k^2 = 2\pi R^2 h/(R+h).$$

The ratio of this area to the area of the spherical earth is $A_Z/4\pi R^2 = h/2(R+h)$, and expressed in percentage is

$$P_1 = 100\, A_Z/4\pi R^2 = 50\, h/(R+h). \qquad (3)$$

The percentages shown in figure 8 were computed from formula (3).

When $\theta + \phi = 90°$, OPS is a right angle and PS is tangent to the earth at P. (see fig. 18). Then the width of a zone about the earth from which a satellite S of altitude h can be seen is given by $2w$ and from (1)

$$2w = 2R\theta, \quad \cos\theta = R/(R+h), \qquad (4)$$

and these were used in preparing curve number 3 in figure 9.

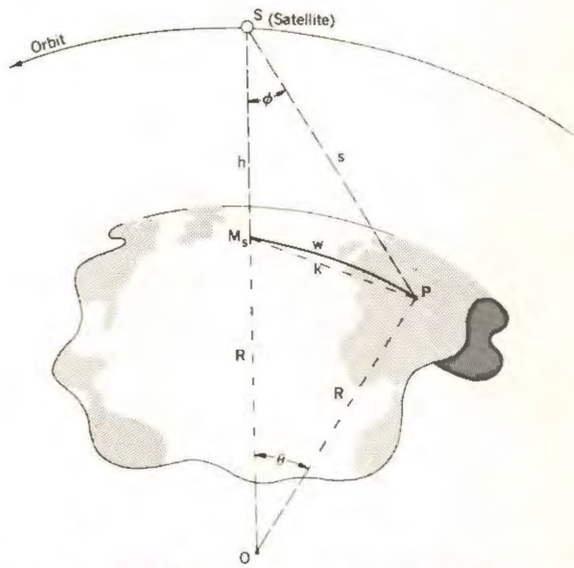

FIG. 18.--Simple geometry of spherical earth-circular satellite orbit configuration.

The total area from which a satellite can be seen during a single transit of its orbit is given by $A = 4\pi R^2 \sin\theta$. Since $A/4\pi R^2 = \sin\theta$, then the percentage of the total area of the surface from which the satellite can be seen during a single transit is

$$P_2 = 100\, A/4\pi R^2 = 100 \sin\theta, \quad \cos\theta = R/(R+h). \qquad (5)$$

Formula (5) was used in preparing figure 8.

The formula for the secular term of the rotation of the orbital plane of a satellite about the earth's polar axis is

$$-\dot{\Omega} = 2\pi \frac{\cos i}{p^2} J,^{23} \qquad (6)$$

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[23]See *Journal of Geophysical Research*, Feb. 1959, page 212; also *U.S.C. & G.S. Tech. Bull. No. 11*.

where $\dot\Omega$ is in radians per period, i is the inclination, $J = 1.6232 \times 10^{-3}$, and for circular orbits, $p = 1 + h/3960$. If the period is T (minutes) and one desires $\dot\Omega$ in degrees per day (24 hours), then (6) may be written

$$-\dot\Omega\,(\text{deg/day}) = \frac{2\pi \cos i}{(1+h/3960)^2}\cdot\frac{180}{\pi}\cdot\frac{1440}{T}\cdot(1.6232)\cdot 10^{-3}$$

$$= (841.467)\cos i \,/\, T(1+h/3960)^2. \qquad (7)$$

Formula (7) was used in preparing figure 10.

APPENDIX 3

THE DOPPLER SHIFT TECHNIQUE

Referring to triangle OSM_1, figure 19, one has by the law of cosines the range s from the ship to the satellite

$$s^2 = R^2 + (R+h)^2 - 2R(R+h)\cos d. \qquad (1)$$

If (1) is differentiated with respect to time and the orbit is assumed circular find

$$s = R(R+h)\sin d\,\dot d/\dot s, \qquad (2)$$

where $\dot d$, $\dot s$ are first derivatives with respect to time. Now the frequency deviation Δf caused by the Doppler effect is

$$\Delta f = -\dot s f_0/c,\quad \dot s = -c\,\Delta f/f_0 \qquad (3)$$

where higher order effects are neglected in view of the low satellite velocity relative to the electromagnetic propagation velocity (c = speed of light, f_0 = satellite transmitter frequency). With the value of $\dot s$ from (3) placed in (2) find

$$s = -f_0 R(R+h)\sin d\,\dot d/c\,\Delta f. \qquad (4)$$

Referring again to figure 19, one has from the spherical right triangle SM_sP

$$\cos d = \cos\theta\cos d_m, \qquad (5)$$

and differentiating (5) with respect to time obtain, since $d_m = 0$ (the rate of change of d_m is zero when perpendicular to the projection of the orbit on earth)

$$\dot d = \cos d_m \sin\theta\,\dot\theta/\sin d. \qquad (6)$$

Placing the value of $\dot d$ from (6) in (4) find

$$s = -\frac{f_0 R(R+h)\cos d_m}{c}\;\frac{\sin\theta\,\dot\theta}{\Delta f} \qquad (7)$$

Now from figure 19 it is seen that when the satellite position M_1 approaches M_m ($M_1 \to M_m$), then $\theta \to 0$, $d \to d_m$, $s \to s_m$, $\dot s \to 0$, $\Delta f \to 0$, $t \to t_m$, and from (7) one has

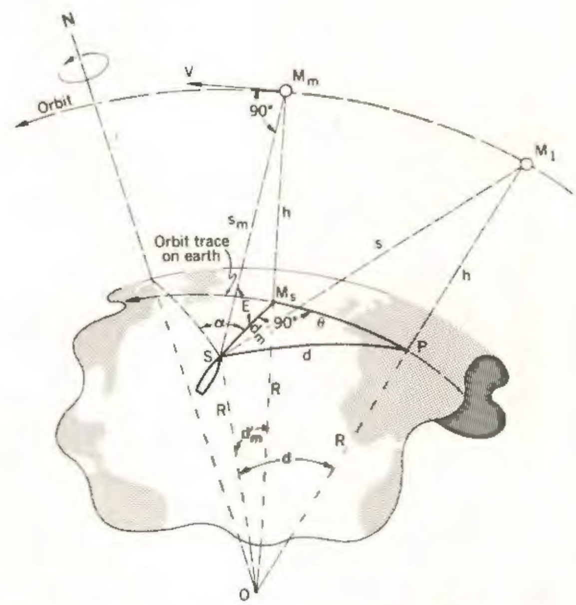

FIG. 19.--Geometry associated with the Doppler shift phenomenon considering a spherical earth and circular satellite orbit at an altitude h.

$$s_m = -\frac{f_0 R(R+h)\cos d_m}{c}\cdot\lim_{\substack{t\to t_m\\ \theta\to 0\\ \Delta f\to 0}}\left(\frac{\sin\theta\,\dot\theta}{\Delta f}\right) \qquad (8)$$

$$\lim\left(\frac{\sin\theta\,\dot\theta}{\Delta f}\right) = \lim_{\substack{\theta\to 0\\ \Delta f\to 0}}\frac{(\cos\theta\,\dot\theta^2 + \sin\theta\,\ddot\theta)}{\dot{\Delta f}} = \frac{\dot\theta^2}{(\dot f)_{max}} \qquad (9)$$

From (8) and (9) one has then

$$s_m = -\frac{f_0 R(R+h)\cos d_m}{c}\cdot\frac{\dot\theta^2}{(\dot f)_{max}}. \qquad (10)$$

The tangential orbital velocity at M_m is $V = \dot\theta(R+h)$ so (10) may be written finally as

$$s_m = -f_0 V^2 R\cos d_m / c(R+h)(\dot f)_{max}$$

and for $d_m < 5°$,

$$s_m = -f_0 R V^2/c(R+h)(\dot f)_{max}. \qquad (11)$$

From triangle OSM_m, figure 19,

$$d'_m = 2 \arcsin \tfrac{1}{2} \left[(s_m^2 - h^2)/R(R+h) \right]^{\frac{1}{2}},$$
$$d_m = R\, d'_m \sin 1''. \tag{12}$$

Hence, since the sensitive radio receiver aboard ship will indicate the instant of maximum frequency change, $(\dot{f})_{max}$, the time t_m of closest approach is obtainable from the corrected ship's chronometer or radio time signals. Then from the satellite ephemeris for the time t_m the tangential velocity V is known, the height h is known, and the geographic coordinates (latitude and longitude) of the subsatellite point M_S. (Of course V and h are constant if the orbit is circular). Since c the velocity of light is known and f_0 the satellite transmitter frequency is known, then the minimum range s_m from the ship to the satellite can be computed from (11). Then d_m can be obtained from equations (12). If h is small compared to R, then one may place $R/(R+h) = 1$ and series may be used for (12), etc.

Finally, if the subsatellite points have been plotted for that particular transit on a conformal map (one which preserves angles), and the scaled distance d_m is projected at right angles to the plotted trace at the particular subsatellite point M_S corresponding to the time t_m, then a position fix for the ship is established. Although a single Doppler observation and reduction will give such a fix, usually a second observation on the same satellite for a second transit or another satellite (with known ephemeris) will be used to give a check.

If at the time t_m of closest approach of the satellite to the ship, the rate of change of azimuth $\dot{\alpha}$ and the angle of elevation E are known (see figure 19), then the following relationships may be derived:

$$\dot{\alpha} = V/d_m, \text{ where } d_m = s_m \cos E$$

whence

$$V = s_m \cos E\, \dot{\alpha}. \tag{13}$$

From (11)

$$V^2 = c\, s_m (R+h)\, \Delta\dot{f} / R f_0, \tag{14}$$

where $\Delta\dot{f} = -(\dot{f})_{max}$.

By dividing respective members of (14) by those of (13) obtain

$$V = c(R+h)\, \Delta\dot{f} / R f_0 \cos E\, \dot{\alpha}. \tag{15}$$

Hence if $\Delta\dot{f} = -(\dot{f})_{max}$ (the maximum rate of Doppler shift) is obtained from the shipboard receiver, and the time t_m of closest approach is determined and for this t_m, $\dot{\alpha}$ and E are known (were measured), with h either known or $(R+h)/R = 1$; then if the tangential velocity V is unknown, it can be computed from (15). Then from (13), d_m and s_m may be computed.

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